1101. Factorise. Show that one of the factors has no real roots (because $x^4$ is non-negative), and give the root produced by the other.

1102. Consider the coefficients of $x^2$, then $x$, then the constant term.

1103. The $x$ inequality produces two regions.

1104. In a large population, the probability that any one datum lies between the quartiles is $\frac{1}{2}$. Selections are being modelled as independent, so you can use the binomial distribution.

1105. Multiply up by both denominators.

1106. The graphs are tangent at $x = a$.

1107. (a) Use similar triangles.
(b) Use calculus: differentiate and set $\frac{dA}{dh} = 0$.

1108. (a) Before adding, redistribute a factor of 10 in the second term to make both indices $n$.
(b) Before adding, redistribute a factor of 10 in the first term to make both indices $n + 1$. You'll need to redistribute another factor of 10 later.

1109. The region is a segment of the unit circle.

1110. Sketch a number line.

1111. Both can exist.

1112. The answer is independent of $k$.

1113. Show that the radii are perpendicular at the points of intersection.

1114. Find $n$ first, by considering the degree of $\mathrm{f}''(x)$. Then differentiate $\mathrm{f}(x)$ twice and factorise.

1115. Assume, for a contradiction, that a pentagon has five interior angles, each of which is smaller than 108°.

1116. (a) Use the first gap, then both gaps together.
(b) Solve simultaneously.

1117. Consider this as a pair of transformations acting on the outputs of the cosine function.

1118. (a) Consider the fact that the test is looking for *negative* correlation.
(b) If the sample statistic is more extreme than the critical value, then the result is significant, which leads to rejection of $H_0$.

1119. This is easiest to visualise in term of replacement of $x$ and replacement of $y$.

1120. In each case, you need the quantity under the square root to be greater than or equal to zero. Set this up as an inequality and solve.

1121. We need to *determine* the roots, as opposed to merely *find* them, so algebraic justification is needed. Use the quadratic formula.

1122. Use the geometry of the situation to find the angle between each section of the line and the horizontal. Then draw a force diagram and resolve vertically.

1123. Find the coordinates of the centre from points $A$ and $B$, then use the fact that $\overrightarrow{CO} = \overrightarrow{OD}$.

1124. Work out the average of the interior angles. Then consider the least possible interior angle. This will allow you to work out the greatest possible interior angle, since an AP is symmetrical about its mean.

1125. Expand the brackets and split up the fraction.

1126. Write this algebraically, then integrate twice.

1127. Express angle as a fraction of a full circle, i.e. of 400 gradians.

1128. You don't need to work graphically: you can solve directly using $|a| = |b| \iff a^2 = b^2$. Having used this, put everything on the LHS and factorise (without multiplying out).

1129. Use a difference of two squares.

1130. Use a Pythagorean identity.

1131. Use the factor theorem: if such an identity exists, then $(2x - 1)$ must be a factor of $2x^2 + 3x + c$.

1132. The curve is a circle. Find its centre.

1133. Set up an equation and solve using logs.

1134. Write $x$ in terms of $u$ and substitute.

1135. Irrespective of the scenario, these forces are always at right angles.

1136. Set up $a = \frac{p}{q}$ and $b = \frac{r}{s}$, for $p, q, r, s \in \mathbb{Z}$.

1137. (a) For $x \leq 0$, $|x| = -x$.
(b) For $x \geq 0$, $|x| = x$.
(c) The graph has a single vertex, like a standard mod graph, but is asymmetrical.

1138. Prove this by contradiction, assuming that f is polynomial of degree $n$. Carry out the integral on the LHS and show that this cannot be equivalent to the RHS.

1139. Use Pythagoras to find the displacement of the point $(x, y)$ for a unit change in the parameter $t$. Then scale this by the length of the $t$ domain.

1140. Split a sector (isosceles triangle) of the $n$-gon up into two right-angled triangles.

1141. Give explicit formulae for $a_n$ and $b_n$. Combine these to make a formula for $c_n$. Identify the first term and common difference.

1142. Find the $x$ coordinate of the vertex of the second parabola. Reflect this in $x = 3$ to find the vertex of the first parabola. This will give you $a$. Reflect the vertex back to find $b$.

1143. (a) Use the change of base formula:

$$\log_a b \equiv \frac{\log_c a}{\log_c b}.$$

(b) Expand the notation and use part (a).

1144. To find the average speed, use $\bar{v} = {}^{\Delta s}/_{t}$. To find the instantaneous speed, use calculus.

1145. Find the distance between the centres of the two circles by Pythagoras, then subtract the radii. The shortest distance lies along the line of centres.

1146. Find the coordinates of the vertices, either using calculus or by completing the square.

1147. These are standard proofs to produce on cue. Use an equilateral triangle of side length 2 and a square of side length 1.

1148. Evaluate the definite integral of $x^2$ between $x = 0$ and $x = 12$. This calculates the total value of function $f(x) = x^2$ over the domain $[0, 12]$.

1149. Find $\dfrac{d^2y}{dx^2}$. Substitute for this and for $y$.

1150. Use logarithms to write $a$ over base 3.

1151. Equate the gradients, as calculated from two pairs of points.

1152. Consider the number of successful outcomes in the possibility space.

1153. Take out a factor of $\sqrt{3}$ first.

1154. Draw a sketch and use Pythagoras.

1155. Sketch the graphs first.

1156. (a) Differentiate twice.

(b) Solve $f''(x) = 0$.

(c) At points of inflection, the second derivative is zero and changes sign. In other words, as the value of $x$ passes through the relevant value, the value of $f''(x)$ passes through zero.

1157. Draw in the radius $OC$. Label the angles at the centre $\alpha$ and $\beta$. Then chase these angles around the triangle to get them to $C$.

1158. Mutual exclusivity implies dependence.

1159. Use the definition ${}^n\mathrm{C}_r = \dfrac{n!}{r!(n-r)!}$.

1160. (a) Start with $\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$.

(b) Use the same method as in (a).

(c) Set $\lambda = \mu = \frac{1}{2}$, and show that the position vectors of $\mathbf{p}$ and $\mathbf{q}$ are the same.

1161. Use the chain rule. A scale factor emerges, because replacement of $x$ by $8x$ has stretched the curve by scale factor $\frac{1}{8}$ in the $x$ direction. This renders all gradients eight times as big as they were.

1162. (a) $(11, 60, 61)$ is a Pythagorean triple.

(b) i. The perpendicular splits $T_2$ into two right-angled triangles. Use Pythagoras on the one with a hypotenuse of length 11.

ii. Use $A_\triangle = \frac{1}{2}bh$.

iii. Use Pythagoras in the other right-angled triangle produced by the perpendicular.

1163. For the vertical asymptote, consider the root of the denominator. For the horizontal asymptote, consider the behaviour for large $x$.

1164. Since $n$ is large, the number of occurrences will be approximately proportional to probability. So, draw the possibility space as a $6 \times 6$ grid, and count equally likely outcomes.

1165. A graph of odd degree must have a shape broadly like that of a cubic: overall (i.e. when moving from large negative $x$ to large positive $x$), it crosses the $x$ axis.

1166. Multiply the number of possibilities for the way out by the number of possibilities for the way back.

1167. Show that $L_2$ and $L_3$ are are reflections of each other in $L_1$, by considering gradients. Reflecting in $y = x$ reciprocates gradients.

1168. Take the cube root of both sides and solve a quadratic in $\sqrt{x}$.

1169. The statement is true.

1170. In each part, quote either NII or NIII.

1171. To find the SPs, set the first derivative to zero. To find the point of inflection, set the second derivative to zero.

1172. Consider the truth of each statement if $x = c$.

1173. Place one vertex, without loss of generality, and consider the possibility space as those remaining.

1174. Consider the degree of the equation satisfied by the intersections.

1175. (a) The curve is a negative parabola.
   (b) Show that the roots of the quadratic are not in the domain of the function.

1176. Expand binomially and simplify. You'll get a cubic in $x^{\frac{1}{2}}$. Factorise this.

1177. The reading on a set of scales is a measurement of reaction force, not mass. The conversion assumes equilibrium, in which case the reaction $R$ is equal to the weight, which is $W = mg$. Draw a force diagram, find $R$, and convert via $R = mg$.

1178. The function g must be quadratic.

1179. The first statement is false.

1180. Call the shorter arc length $l$. In terms of $l$, find the radius of the smaller circle, then the radius of the larger circle, then the length of the longer arc. Using the perimeter, set up an equation for $l$ and solve.

1181. The limit is well defined, the evaluation is not.

1182. (a) Rearrange $a_1b_1 + a_2b_2 = 0$ to show that the gradients of **a** and **b** are negative reciprocals.
   (b) The vector $\mathbf{a} - \mathbf{b}$ is the vector from point $B$ to point $A$. You know the lengths of **a** and **b**. Use Pythagoras to find $|AB|$.

1183. Remember that mass is fixed in the Newtonian system, but weight is not.

1184. The notation $\text{f}^2(x)$ means f applied twice to $x$, i.e. $\text{f}(\text{f}(x))$. So, for a counterexample, you need to find two functions which, when applied twice, have the same effect as each other, but when applied once, don't.

1185. (a) Use Pythagoras.
   (b) Consider the hexagon as a set of six equilateral triangles.

1186. One of the statements is false. The point is that *every* element of the range must be attainable as an output by the function.

1187. Use the same technique as is used for rationalising the denominator of surds. Multiply top and bottom by the conjugate of the top.

1188. Visualise/draw the sets on a number line. In each case, the boundaries of the left-hand set are $\pm 2$.

1189. To answer parts (a) and (b), complete the square on both parabolae.

1190. (a) Draw a force diagram to calculate $a$, and then use *suvat* to find the height and speed at which the fuel runs out.
   (b) To find the displacement during the second stage, set $v = 0$ in $v^2 = u^2 + 2as$. Use the final velocity of the first stage as the initial velocity of the second stage.

1191. Subtract the areas of three isosceles triangles from the area of the square.

1192. (a) Consider the necessary value of $2x$.
   (b) If two numbers multiply to give zero, then...

1193. Engineer a counterexample in the form $a$ and $a+b$.

1194. Work graphically: thinking of the equations as straight lines, compare the gradients.

1195. Calculate the two integrals explicitly, noting that the second is an integral with respect to $y$.

1196. Consider the difference and sum of the roots, as given by the quadratic formula. Alternatively, set up and solve a pair of simultaneous equations for the roots themselves.

1197. (a) Equate the coefficients of **a** and the coefficients of **b**. Solve simultaneously.
   (b) Work out precisely which step of the argument would have broken down if
      i. the vectors were parallel,
      ii. one of the vectors were zero.

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1198. (a) Consider the number of steps required to get to the $n$th term.

 (b) Consider the mean of the sequence, which is the mean of the first and last terms.

1199. Consider the expression as a gradient.

1200. Use either elimination or substitution.

——— End of 12th Hundred ———